Gravitational Interaction of Neutrinos in Models with Large Extra Dimensions

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Abstract. Whenever fields are allowed to propagate in different portions of space-time, the four dimensional theory exhibits an effective violation of the principle of equivalence. We discuss the conditions under which such an effect is relevant for neutrino physics. In the simplest case of compactification on a flat manifold, the effect of gravity is many orders of magnitude too weak and plays no role for solar neutrino oscillation. Instead, it could be important in the study of ultra high-energy neutrinos in cosmic rays. Gravity could also be relevant for lower energy neutrino processes involving bulk sterile states, if the mechanism of compactification is more subtle than that on torii.

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1 Introduction

Neutrinos are neutrinos because they only interact by means of weak interactions and gravity. The latter interaction is usually negligible because of its intrinsic weakness and the universal way it acts on all neutrino species.

Attempts to bring gravity into play in neutrino physics have necessarily assumed some form of explicit violation of the equivalence principle [1]. The violation is introduced by hand within a somehow vague theoretical framework; the absence of an action from which to derive the equations of motion may lead, among other problems, to energy non-conservation.

Quite independently of these efforts, in a parallel development, models with space dimensions in addition to the usual three—and large enough to be observable even after compactification—have been suggested [2] as a possible solution to the problem of the large scale difference between gravity and the standard model. In these models the standard model lives in four-dimensional space time [3] and—along the lines of brane-world models [4]—only gravity inhabits the extra dimensions. The strength of gravity is re-scaled up to a value not far from that of the Fermi constant, and the experimental weakness of the two body potential of gravity is explained by the space volume of the large extra dimensions, for which the gravitational coupling at large distances must be divided.

In addition to gravity also matter can be allowed to propagate in the extra dimensions. In particular, bulk (sterile) neutrinos have been suggested and their interaction with the standard-model neutrinos discussed [5,6]. In these models, bulk neutrinos are coupled to the standard-model neutrinos to give the latter mass and the the ensuing mixing was argued to be consistent with solar, atmospheric and other neutrino experimental data involving sterile fermions, before the advent of SNO results [7].

However, the interplay between matter and gravity in the extra dimensions had not been considered. Since gravity is brought down to a much smaller energy scale, it is natural to ask whether its effect on neutrino physics may become as important as that of other forces. In this letter we show that for certain choices of energy and compactification scales, the effect can be sizable. This effect is not a mere common renormalization because particles confined within four dimensions and those allowed to propagate in the extra dimensions feel a different gravitational interaction. As we shall see, even though no violation of the equivalence principle is assumed in the fundamental theory (and therefore there is no problem with energy conservation), the different shapes of the wave functions in the extra dimensions produce an effective (as opposed to an explicit) violation in the four-dimensional theory. This means that the model provides a consistent framework to discuss effective violations of the equivalence principle in neutrino physics along the phenomenological lines of [1].

2 Sterile neutrinos in the bulk

Let us consider space-time to consist of the usual four dimensional Minkoski space plus δ space-like extra dimensions, describing a δ-dimensional compact manifold. For instance, the simplest possibility is that the compact manifold is a torus, with all the extra dimensions describing a circle with radius R, small enough to escape experimental observation.

We only consider one of the extra (compact) dimensions to be large enough to have observable effects, while the others are at much smaller scales —as we shall see, for more than one large extra dimension the power law dependence of gravitational effects on neutrinos render them irrelevant—and take its size to be $R \sim 100$ μm (that is, $1/R \sim 2 \times 10^{-3}$eV) which is relevant for the experimental tests of sub-millimeter gravity and satisfies current upper bounds [8].

Most of our discussion will be, for the sake of definiteness, about a model [6] in which all standard-model fields are assumed to be localized within 4-dimensional space-time (for instance on a 3-brane), while a fermionic massless standard-model singlet is allowed to propagate in the bulk with gravity. The Yukawa coupling of left-handed neutrinos with the sterile neutrino provides a Dirac mass term $m^{(5)}$ through the Higgs mechanism. This mass must be tuned in such a way that the coupling neutrino-bulk fermion is in the correct range in order to reproduce the neutrino oscillation phenomenology [6].

Once the Kaluza-Klein (KK) expansion has been performed, we are left with a tower of sterile neutrinos in the four-dimensional theory—with masses given by an integer multiplied by $1/R$ —coupled to the standard neutrino via the Dirac mass term m_D. The phenomenological analysis performed in [6] shows that this model, for suitable values of the four dimensional Dirac mass term M_D, lies in the correct range to reproduce the Mikheyev-Smirnov-Wolfenstein (MSW) Small Mixing Angle (SMA) solution to the solar-neutrino deficit, thus providing an elegant solution to the hierarchy problem.

The SMA-MSW solution is not compatible with the recent data from SNO collaboration [7], and also the possibility that subleading oscillations involving sterile neutrinos take place seems disfavored from the combined analisis of all neutrino experiments [9]. Nevertheless, the model in [6] is still of interest for its minimality and simplicity. For these reasons, we will use it as a toy-model for illustrative porposes, since it allows a direct comparison between the magnitude of the gravitational and weak intereaction effects.

3 Matter effects: weak vs. gravitational

Weak interactions of neutrinos with matter are not universal: charged leptons in ordinary matter have all the same flavor (that of the electron), and therefore charged current interactions of neutrinos going through a medium like the sun or the earth distinguish the first family neutrinos ν_L^e from the other species. Consequently, the Dirac equation of electron neutrinos propagating in the medium contains a potential (energy) term of the

$$V_m = \sqrt{2}\, G_F\, \xi_e \,, \tag{1}$$

with ξ_e electrons per unit volume.

The oscillation phenomenon is then determined not only by the mass-squared differences and vacuum mixing angles between the flavors, but also by the density of the medium, which modifies the effective mixing angles between ν_L^e and the other species. In the two neutrino flavor oscillation approximation, the mixing angle in matter θ_m is given by

$$\tan 2\,\theta_m = \frac{\sin 2\,\theta_0}{\cos 2\,\theta_0 - 2\,p\,V_m\,(\Delta m)^{-2}} \,, \tag{2}$$

where p is the energy of the neutrinos (ultra-relativistic approximation), while Δm^2 and θ_0 are the oscillation parameters in vacuum. The minus sign in the denominator of Eq. (2) allows for the possibility of a resonant transition, leading to the MSW effect [10].

On the other hand, neutral current interactions do not affect the oscillation because of their universality (in flavor space): a possible contribution V_m' to the Hamiltonian of the system is there, but it can be factorized out from the Dirac equation. This flavor universality is shared also by ordinary gravitational interactions thus making its detection impossible.

Several proposals have been put forward in the past to violate flavor universality by means of a small violation of the equivalence principle [1]. This fact is achieved by the addition by hand of flavor violating couplings to gravity, which give rise to a gravitational potential term V_G in the evolution equations, and thus modifying the mixing angles in matter.

All these proposals come with some unwelcome feature like massive gravitons or energy non-conservation because of the explicit violation of the principle of equivalence. For this reason, it would be desirable to explore models in which the equivalence principle is not violated explicitly in the action even though gravity couples differently to different kinds of neutrinos. This happens in some models based on the existence of more than four space-time dimensions. In this case, the principle of equivalence applies to the full D dimensional system, but while matter and other standard model fields are localized on a four-dimensional manifold, new degrees of freedom can propagate in the whole space, and therefore their gravitational coupling to matter can be different from those of standard neutrinos. ¿From the four dimensional effective theory point of view, this shows up as an explicit violation of the universality of gravitational interaction: the couplings to gravitons are different for standard model fields and the KK modes of the new, higher dimensional degrees of freedom.

Gravitational effects of this type are potentially at work in all models where fermions are introduced in the bulk to reproduce the phenomenology of the oscillation between SM and sterile neutrinos. Moreover, in these models, the strenght of the gravitational coupling, that rules this effect, is not a priori negligible because higher dimensional effects can render it larger.

In order to show explicitly how this arises, and quantify its impact on model building, we must compute the forward scattering amplitude between matter and neutrinos which lead to Eq. (1), substituting the exchange of a W boson with a graviton. Let us consider as an illustrative example the case $D = 5$ with a flat background, with one extra-dimension compactified on a circle of radius R.

In this case, the interaction is given by

$$\sqrt{4\,\pi^2\,G^{(5)}}\,h_{\mu\nu}\,T^{\mu\nu}\,, \tag{3}$$

where the 5-dimensional metric tensor is defined as $g_{\mu\nu} = \eta_{\mu\nu} + 2\sqrt{4\,\pi^2\,G^{(5)}}h_{\mu\nu}$, while $T^{\mu\nu}$ is the Dirac field energy-momentum tensor in flat space-time, and

$$G^{(5)} \equiv 1/(2\,\pi\,M_5^3) \tag{4}$$

replaces G_N (Newton constant) thus changing the strength of gravitational interactions. We consider an idealized situation in which gravity behaves as a five dimensional theory up to a distance equal to $\rho \leq R$, and returns back to the usual four dimensional, very weak interaction for distances grater than ρ.

The final integration over the 4-dimensional space, having inserted the correct form of the graviton two-point correlator, gives

$$\begin{aligned}\int \frac{\mathrm{d}^3x'\,\mathrm{d}y'\,\delta(y')}{|x'-x|^2+|y'-y|^2} &= \int_{3-\mathrm{brane}} \frac{\mathrm{d}^3x'}{|x'-x|^2+y^2} \\ &= 4\,\pi\left\{\sqrt{\rho^2-y^2} - y\arctan\sqrt{\rho^2-y^2}/y\right\}\,.\end{aligned} \tag{5}$$

The δ-function in the extra dimension y' is there because ordinary matter (which we take as source of the gravitational field) is constrained within the 3-dimensional space. We use the approximation of letting the gravitational potential act only up to distances of order ρ, since the contribution coming from an integration on larger distances is negligible.

Finally:

$$\begin{aligned}V_G(y) = &-8\,\pi^2\,G^{(5)}\,p\,\xi_N\,m_N \times \\ &\times \left\{\sqrt{\rho^2-y^2} - y\arctan\sqrt{\rho^2-y^2}/y\right\}\end{aligned} \tag{6}$$

The mass m_N is the rest energy of the nucleons (of about 1 GeV), while ξ_N is their number density in the medium. Notice in Eq. (6) the extra energy dependence with respect to Eq. (1).

The y-dependence of the potential is crucial. For a system of two neutrinos (like ν_e and ν_μ) , both constrained inside our 3-dimensional space, the potential V_G is only felt at $y=0$, thus giving a common factor

$$\int V_G(y)\,\delta(y)\,\mathrm{d}y = V_G(0) \tag{7}$$

that can be rotated away in the evolution equation: the principle of equivalence is here at work. On the other hand, bulk neutrinos propagate in the extra dimensions and feel the whole potential; accordingly their gravitational interaction is different from that of ordinary neutrinos.

The Dirac equations for the standard ν_L^e and bulk neutrinos, arbitrarily denoted as $N_{L,R}$, in five dimensions (and with the γ matrices in chiral representation) propagating in matter of constant density become:

$$\left\{\begin{aligned}\Big[\partial_0 + \sigma^i\partial_i + i\,V_m + i\,V_G(y)\Big]\nu_L &= -i\,m^{(5)}N_R \\ \Big[\partial_0 + \sigma^i\partial_i + i\,V_G(y)\Big]N_L &= -\partial_y N_R \\ \Big[\partial_0 + \sigma^i\partial_i + i\,V_G(y)\Big]N_R &= \partial_y N_L + i\,m^{(5)}\nu_L\,,\end{aligned}\right. \tag{8}$$

where y represents the 4-th space-like coordinate.

4 Neutrino oscillations

The evolution equations for the neutrino system, containing the new contributions from gravitational interactions, lead to a modified expression for the mixing angles between standard model neutrino and the tower of KK modes of the five dimensional field. Let us expand the five dimensional field N_L in a Fourier series on the fifth coordinate. The field N_R decouples from the system and we will not consider it any more. The evolution equations of the system can be studied in terms of the KK modes, and we can write it as

$$i\frac{\mathrm{d}}{\mathrm{d}t}\begin{pmatrix}\nu_L\\ N_L^{(1)}\\ N_L^{(-1)}\\ \cdots\\ N_L^{(n)}\\ N_L^{(-n)}\end{pmatrix} = \mathcal{H}\begin{pmatrix}\nu_L\\ N_L^{(1)}\\ N_L^{(-1)}\\ \cdots\\ N_L^{(n)}\\ N_L^{(-n)}\end{pmatrix}\,, \tag{9}$$

where $\mathcal{H}$ is defined in Eq. (10).

In Eq. (9) we left out common (kinetic and potential) terms that appear in the diagonal of the matrix Eq. (10), and contributions quadratic in the potentials. In Eq. (10),

$$V_G^{(n)} \equiv \frac{1}{2R}\int_{-R}^{+R}\cos\left(\pi n y/R\right)V_G(y)\,\mathrm{d}y \tag{11}$$

is the n-th Fourier component of the gravitational potential energy.

To gauge the magnitude of this new gravitational term, we have written in Eq. (10) the difference between the potential in $y=0$ and its zero mode as

$$V_{\mathrm{G}}(0) - V_{\mathrm{G}}^{(0)} \equiv -\Omega\,V_{\mathrm{m}}\,, \tag{12}$$

with

$$\Omega \simeq 10^5\,\left(\frac{1\,\mathrm{TeV}}{M_5}\right)^3\left(\frac{\rho}{100\,\mu\mathrm{m}}\right)\left(\frac{p}{1\,\mathrm{MeV}}\right)\frac{\xi_N}{\xi_e}\,. \tag{13}$$

For a situation in which $V_m = V_G^{(n)} = 0$ in Eq. (10), the oscillation in vacuum between the standard-model neutrino and the n-th state $N^{(n)}$ of the tower of KK sterile neutrinos, with masses given by n/R, is governed by the mixing angle

$$\tan 2\,\theta_0^{(n)} = \frac{2\,m_D\,(n/R)}{(n/R)^2 - m_D^2(n+1)}\,. \tag{14}$$

$$2\,p\,\mathcal{H} = \begin{pmatrix} m_D^2(n+1) - 2p(1-\Omega)V_m & m_D/R & -(m_D/R) & 2(m_D/R) & -2(m_D/R) & \cdots & n(m_D/R) & -n(m_D/R) \\ (m_D/R) & 1/R^2 & 2pV_G^{(2)} & 2pV_G^{(1)} & 2pV_G^{(3)} & \cdots & 2pV_G^{(n-1)} & 2pV_G^{(n+1)} \\ -(m_D/R) & 2pV_G^{(2)} & 1/R^2 & 2pV_G^{(3)} & 2pV_G^{(1)} & \cdots & \cdots & \cdots \\ 2(m_D/R) & 2pV_G^{(1)} & 2pV_G^{(3)} & 4/R^2 & 2pV_G^{(4)} & \cdots & \cdots & \cdots \\ -2(m_D/R) & 2pV_G^{(3)} & 2pV_G^{(1)} & 2pV_G^{(4)} & 4/R^2 & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ n(m_D/R) & 2pV_G^{(n-1)} & \cdots & \cdots & \cdots & \cdots & n^2/R^2 & 2pV_G^{(2n)} \\ -n(m_D/R) & 2pV_G^{(n+1)} & \cdots & \cdots & \cdots & \cdots & 2pV_G^{(2n)} & n^2/R^2 \end{pmatrix} . \qquad (10)$$

Consider now the effect of matter, but neglecting for a moment gravitional interactions. As neutrinos enter space filled by matter with constant density, the mixing angle becomes

$$\tan 2\,\theta_m^{(n)} = \frac{\sin 2\,\theta_0^{(n)}}{\cos 2\,\theta_0^{(n)} - 2\,p\,V_m\,(R/n)^2} . \qquad (15)$$

The weak potential modifies the mixing angle, leading to a possible resonance, in a way that strongly resembles the usual MSW effect.

Let us finally switch on gravity in our system. Neglecting for the moment off-diagonal terms in Eq. (10) originating from higher Fourier modes of the gravitational potential, we can write the mixing angle as

$$\tan 2\,\theta_{(m+G)}^{(n)} = \frac{\sin 2\,\theta_0^{(n)}}{\cos 2\,\theta_0^{(n)} - 2\,(1-\Omega)\,p\,V_m\,(R/n)^2} . \qquad (16)$$

If the sterile neutrinos were constrained on the four dimensional world, Ω, as well as all the coefficients $V_G^{(n)}$, would vanish and there would be no effect. If $\Omega \gg 1$, the effective mixing angle in matter of ν_L and, for instance, the first KK mode $N^{(1)}$ is suppressed by the factor Ω^{-1}:

$$\tan 2\,\theta_{(m+G)}^{(1)} \sim \Omega^{-1} \tan 2\,\theta_m^{(1)} . \qquad (17)$$

Accordingly, gravity could decouple the two neutrino modes. Let us stress that the sign of the gravitational potential is opposite to the weak potential in Eq. (1): when V_G is large enough to flip the sign of matter effects, it is no more possible to satisfy a resonance condition, since we consider small mixing angles θ_0.

This fact allows also for the possibility of a compensation, for a choice of parameters that leads to $\Omega \simeq 1$, between weak and gravitational contributions. In this case, even in presence of matter, the effective mixing angles result the same as in vacuum.

Let us stress an important difference between gravitational and weak interactions: while the former are the same for particles and antiparticles, the presence of a medium makes the latter act differently between neutrinos and antineutrinos. In this sense, if gravitational effects are relevant, they can be factorized in a comparative study between neutrino and antineutrino fluxes, and it is possible to extract indepent information on weak and gravitational effects.

The same results hold by using the full matrix (10), off-diagonal Fourier terms included. We have checked numerically that the diagonalization of the matrix, for the values of Ω discussed above, gives rise to eigenvectors in which ν_L is, for all practical purposes, decoupled from the bulk-neutrino modes.

5 Model dependent considerations

Having established that gravity does affect neutrino oscillations in models where the sterile neutrinos propagate through the extra dimensions, let us now discuss the possible relevance of this effect.

Our discussion can be seen from two complementary points of view: either as the search for models in which gravitational effects are important or as the definition of models which are safe from such effects. Indeed, even showing that gravity—although present— does not significantly modify the physics is relevant, since most neutrino physics is valuable exactly because neutrinos only interact via electroweak forces.

The first example we consider is that of solar neutrinos in the $D = 5$ model with one extra-dimension compactified on a circle of radius R [2] that we used as a template in the previous sections. In this case, we can use directly the computation performed in Section (3), identifying $\rho \simeq R$. The four dimensional effective Planck scale $M_{\rm Pl}$ is connected with the foudamental scale M_5 by

$$M_{\rm Pl}^2 = 2\,R\,M_5^3 . \qquad (18)$$

The largest allowed radius is at the sub-millimiter scale, so that $M_5 \sim 10^5$ TeV.

Considering typical energies for neutrinos $p \sim$ few MeV, and an averaged value for the solar density, we obtain for Ω the value 10^{-10}. This means that for all practical purposes, in Eq. (10) one can set to zero V_G^n and Ω: gravity is negligible in respect to the weak interaction effects, even in presence of sterile KK states.

How about the more general case of models based on more complicated space-times? One should repeat the computation of Section 3, substituting G_5 in Eq. (3) with

the new coupling, and the appropriated graviton two-point correlator in Eq. (5).

For two flat extra dimensions of the same size, the two point correlator goes like r^{-3} instead of r^{-2}. This implies that the potential in Eq. (5) yields a logarithmic dependence on R which makes the effect smaller. Further additional dimensions make the potential even weaker. In this case, even phenomena involving ultra high-energy neutrinos are not affected.

Another case in which the two point correlator is known is the Randall-Sundrum model [3]: the power law dependence of the gravitational potentials are similar to the one of $D = 6$ ADD model, and the same considerations hold, even if the fundamental coupling of the model is lowered with respect to the original idea of RS, in which Planck scale gravity inhabits the bulk.

We therefore conclude that, at least for these models for which we know how to compute the gravitational potential, we do not expect gravity to play a role at the energies of solar neutrinos, because of the large value of the scale M_5 suppresses their gravitational interactions.

However, even for M_5 of the order of 10^5 TeV, there could be important effects for neutrino physics in other situations. Consider, for instance, ultra-high energy neutrinos produced by astrophysical objects like gamma ray bursters or active galactic nuclei: their oscillations, also to sterile neutrinos, in presence of matter effects have been discussed [11]. If any sterile neutrino lives in the bulk, a Dirac equation similar to the one discussed for the solar neutrinos applies, in which matter effects are due to the interstallar medium. The suppression in Eq. (13) coming from the large value of M_5 is compensated, in this case, by the energy p which can be as large as 10^9 TeV, even if the densities are much smaller: the gravitational terms effectively compete with weak interactions in determining matter effects in the oscillations.

Possible gravitational interactions of ultra-high energy neutrinos has also been discussed for the physics of scattering processes in [12].

6 Model independent considerations

The actual value of M_5 in a generic model should be fixed by the dynamics of the system which gives rise to the compactification geometry and by the implied relation between the effective extra dimensional couplings and G_N. This is an open problem in extra-dimensional models, and, in general, M_5 is neither given by Eq. (18) nor known, apart from few peculiar examples, as the RS model.

For this reason it is also useful to follow a purely phenomenological approach in which ρ and M_5 are only restricted by the experimental bounds. We assume un-suppressed gravity to be localized at very short distances, smaller than ρ, while for $r \gg \rho$ the usual Newtonian regime (up to subleading long-range corrections, due to the exchange of heavy KK states) is recovered.

In general, from the combined gravimetrics [8] and particle-physics [13] experimental bounds we can extract the scale M_5 and the effective range ρ of gravity which we use as the cut off in Eq. (5). Since these bounds are crucial, we summarize and discuss them in the following subsections.

6.1 Searches for non-newtonian gravity

While Newtonian gravity above the centimeter is well confirmed [14], its short distance behavior is still under active scrutiny. All experiments, regardless of the actual apparatus, set a bound on non-Newtonian interactions from the absence of deviations between the force measured at distance r^* and the predicted one.

The bound is usually given in terms of the parameters α and λ according to the two-body potential

$$V(r)|_{r^*} = \frac{G_N m_1 m_2}{r} \left[1 + \alpha_G \, e^{-r/\lambda} \right] \Bigg|_{r^*} , \tag{19}$$

where G_N is the Newton constant in four space-time dimensions. Because of the exponential behavior, the best sensitivity is achieved in the range $\lambda \sim r^*$. Currently, experiments testing Van der Waals forces are sensitive to the range $r^* \sim 1.5 \div 130$ nm [15]; Casimir-force experiments explore $r^* \sim 0.02 \div 6$ μm , r^* being here the distance between dielectrics or metal surfaces [16,17] or up to mm by means of a torsion pendulum[18]. Cavendish-type experiments, in which the gravitation force is directly measured, are sensitive to $r^* > 1$ mm [19].

The exclusion regions thus determined are convex curves around the distance r^* at which the experiment is performed. The sensitivity of the experiments rapidly decreases at smaller wave-lenghts. The combined exclusion regions obtained by these searches, for the relevant distances, are shown as grey areas delimited by black curves in Fig. 1.

6.2 Gravitational potential in models with large extra dimensions.

The two-body potential in models with δ flat extra dimensions can be parameterized (for r less than R^*, the characteristic compactification length) as

$$V_\delta(r) = \frac{G_N m_1 m_2}{r} \left(\frac{a_\delta}{r} \right)^\delta . \tag{20}$$

In Eq. (20)

$$a_\delta = (G^{(\delta)}/G_N)^{1/\delta} = \frac{2\,\pi}{M_f} \left(\frac{4\,\pi}{\Omega_\delta} \frac{M_{\rm P}^2}{M_f^2} \right)^{1/\delta} , \tag{21}$$

where we define $M_{\rm P} \equiv 1/\sqrt{G_N} = 1.22 \times 10^{16}$ TeV. In Eq. (21), $\Omega_\delta = 2\pi^{(3+\delta)/2}/\Gamma[(3+\delta)/2]$ and M_f is the scale of the effective theory. For distances larger than R^*, the potential in Eq. (20) is replaced by the usual Newtonian potential plus exponentially small corrections:

$$V_\delta(r) = \frac{G_N m_1 m_2}{r} \left[1 + \alpha_\delta \, e^{-r/R^*} + \cdots \right] . \tag{22}$$

In Eq. (22), the value of α_δ depends on the compactification choice and is of the order of the number of extra dimensions [23].

It is important to bear in mind that Eq. (22) depends on the way the extra dimensions are treated in the process of compactification while Eq. (20) only relies on Gauss law and is therefore compactification independent.

When the experimental bounds parameterized by Eq. (19) are plotted (on a logarithmic scale in the $(\alpha - \lambda)$-plane) against Eq. (22), a single point is obtained at $\alpha_G = \alpha_\delta$ and $\lambda = R^*$. For instance, models with extra-dimension compactified on torii with equal radii R^* predict $\alpha_\delta = 2\delta$. This leads to the known bound $R^* \gtrsim$ few 10^{-4} m, as can be deduced by Fig. 1, with $R^* = \lambda$.

6.3 Compactification-independent bounds from particle physics.

Contrarily to short-distance gravity measurements, particle-physics measurements only constrain the effective gravitational coupling $G^{(\delta)}$ by means of the bound on M_f. The independence from r is manifest in the $(4+\delta)$-dimensional theory, which probes distances much smaller than the compactification radius R^*, and recovered in the 4-dimensional computation after resumming over the Kaluza-Klein states.

For this reason, the relationship obtained by comparing Eq. (19) and Eq. (20), must be valid for any choice of r (as long as $r \lesssim R^*$) and gives the stringiest bound at the minimum. Therefore, the curve of exclusion is found to be

$$\alpha_G(\lambda) \le \min_r \left\{ \left[\left(\frac{a_\delta}{r} \right)^\delta - 1 \right] e^{r/\lambda} \right\}. \qquad (23)$$

To find the curve given by Eq. (23), we must solve the polynomial equations obtained by the minimalization procedure. Exact solutions exist for $\delta < 4$; however, for all practical purposes, approximated solutions can be found by elementary calculus for any δ. The exclusion region is given by the lines

$$\alpha_G(\lambda) = \begin{cases} \left[(a_\delta/\delta\lambda)^\delta - 1 \right] e^\delta & \text{for } \lambda < \lambda_{max} \\ \alpha_{min} \equiv \delta\, e^{\delta+1} & \text{for } \lambda \ge \lambda_{max} \,, \end{cases} \qquad (24)$$

where $\lambda_{max} = (1+\delta)^{-(1+\delta)/\delta} a_\delta$. The value λ_{max} is reached when no real solution can be found. The exclusion region is extended for $\lambda > \lambda_{max}$ by taking smaller values of M_f (already excluded) for which the solution is translated to larger values of λ while still ending at the same (constant) value of α_{min}.

In collider physics, the most effective channel at both LEP and Tevatron is that in which virtual gravitons take part in dilepton or diphoton production. Production of real graviton gives less stringent bounds. Whenever the bound depends on the sign of the potential we have taken the lesser bound. Recent reviews of all these bounds can be found in Ref. [24].

We have summarized in Table 1 the best bounds from particle physics.

Table 1. Particle physics bounds on M_f. The numbers reported are the constrains in TeV for the first few large extra dimensions δ. Missing entries were not reported in the literature.

	δ			
measurement	1	2	3	reference
LEP	1.2	1.2	1.2	[20]
Tevatron I	-	1.5	1.5	[21,22]
Tevatron IIb	-	3.5	3.0	[22]
LHC	-	13	12	[22]

While precision measurements and collider bounds from production of real gravitons depend on the number of extra dimensions, those from virtual graviton processes at colliders are (almost, for certain parameterizations) independent. Bounds from oblique parameters are potentially very restrictive but are plagued by infrared divergences which make the final result rather uncertain [25]. For this reason we will not use them.

Even though a degree of uncertainty remains in these calculations because of the cut-off dependence (and because of different parameterizations), the bounds work on order of magnitudes and are therefore sufficiently reliable as they stand. In particular, we neglect small discrepancies between different approaches [26].

We keep in Table 1 also the δ =1 case, even though it is often considered ruled out. This is true only after having assumed a specific compactification geometry and we want to use the particle-physics constraints irrespectively of this additional assumption.

Given the particle-physics bounds in Table 1 and Eq. (24), we obtain the curves in Fig. 1, where the respective exclusion regions (the area above the lines) are presented for the first few extra dimensions.

In using these bounds, the values for α_G and λ of a specific model must be plotted against the bounds of the corresponding effective theory at $r \sim \lambda$, the space dimension of which is not necessarily that of the fundamental theory.

Figure 1 shows that for $\delta > 2$ particle-physics bounds, in particular those coming from collider physics, are various orders of magnitude stronger than direct searches for non-Newtonian gravity below the mm. In other words, if any deviation is ever found in these experiments, it will not be possible to explain it in terms of large extra-dimension models.

On the contrary, for $\delta = 1$ in the range $\lambda \gtrsim 1$ nm Casimir and Cavendish-like experiments are the most sensitive and rule out a large amount of parameter space, while particle physics is relevant only at much shorter distances. Notice that the bounds still allow a strong gravity coupling (of the order of $1/(\mathrm{TeV})^3$) up to few nm as long as it then decreases fast enough to match the long distance regime, in order to satisfy bounds from gravimetric experiments.

The conclusion of this analysis of the experimental bounds is that, for most of the range of parameters we are interested in, high-energy experiments give the strongest

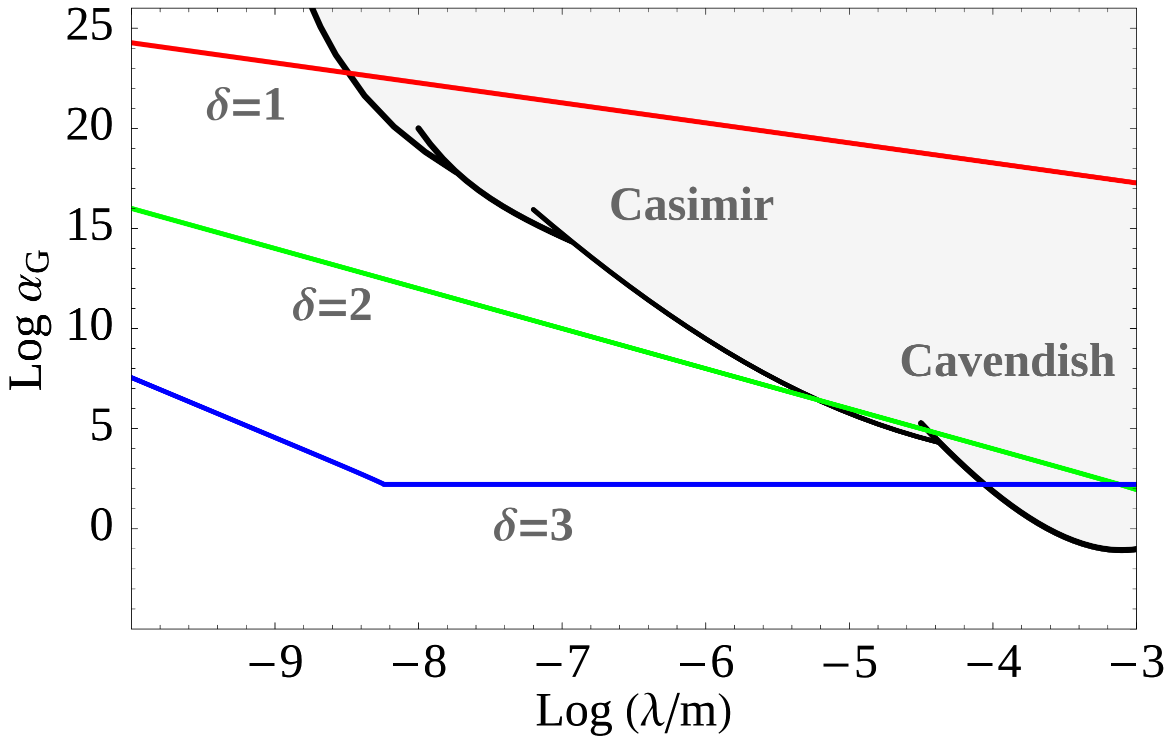

Fig. 1. Bounds α_G vs. λ for $\delta = 1$, 2 and 3 from current particle-physics tests (see Table 1). The thicker curved lines are the best available bound from non-Newtonian gravity experiments and the grey area is the range excluded by them.

bounds on the size of space-time extra dimensions and gravity strength. Coming back now to the impact of gravity on neutrino physics, even by taking into account these bounds, and therefore by considering distances shorter than $\rho \sim 10^{-3}\mu$m and taking the limit of $M_5 \geq 1.2$ TeV, the crucial factor Ω in (13) can be as large as ten for neutrino energies in the range of solar neutrino physics.

7 Conclusions

We have shown that whenever fields with the same quantum numbers are allowed to propagate in different portions of space-time, the four dimensional theory exhibits an effective violation of the principle of equivalence. This implies that, in principle, gravity could play a role in flavor violating phenomena such as neutrino oscillations in matter, due to the non-universality of the gravitational coupling.

Furthermore, large extra dimension scenarios could imply a strong enhancement of the coupling at short distances, where the space-time is effectively higher dimensional. We focused on the case of just one extra dimension, and discussed explicitly the conditions under which such an effect is relevant for neutrino physics, even though our discussion is only at the level of orders of magnitude and matter is taken to have a constant density distribution.

Although the model we considered is not compatible with recent experimental results, its simplicity allowed us to show how to estimate quantitatively possible effects of gravitational interactions in neutrino experiments.

Accordingly, the presence of the gravitational potential may, in principle, drastically change the values of the parameters used in fitting the experimental data and the interpretation of a resonance solution; moreover, it can produce a peculiar distortion of the neutrino energy spectrum because of the extra energy dependence.

In the simplest case of compactification on a flat manifold, the effect of gravity is many orders of magnitude too weak for observation, and only weak interactions play an important role for solar neutrino oscillation. Still, gravity could be relevant for more exotic phenomena, for example in the study of ultra high energy neutrinos in cosmic rays, where the effect is enhanced by the peculiar energy dependence of gravitational interactions. In this case, the generic effect of gravitational interactions is to suppress the effective mixing angles in matter.

To conclude, let us notice that gravity could also be relevant for sterile neutrino physics at lower energies if the mechanism of compactification is more subtle than that on torii, and provided all the bounds on sterile neutrino physics are satisfied.

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